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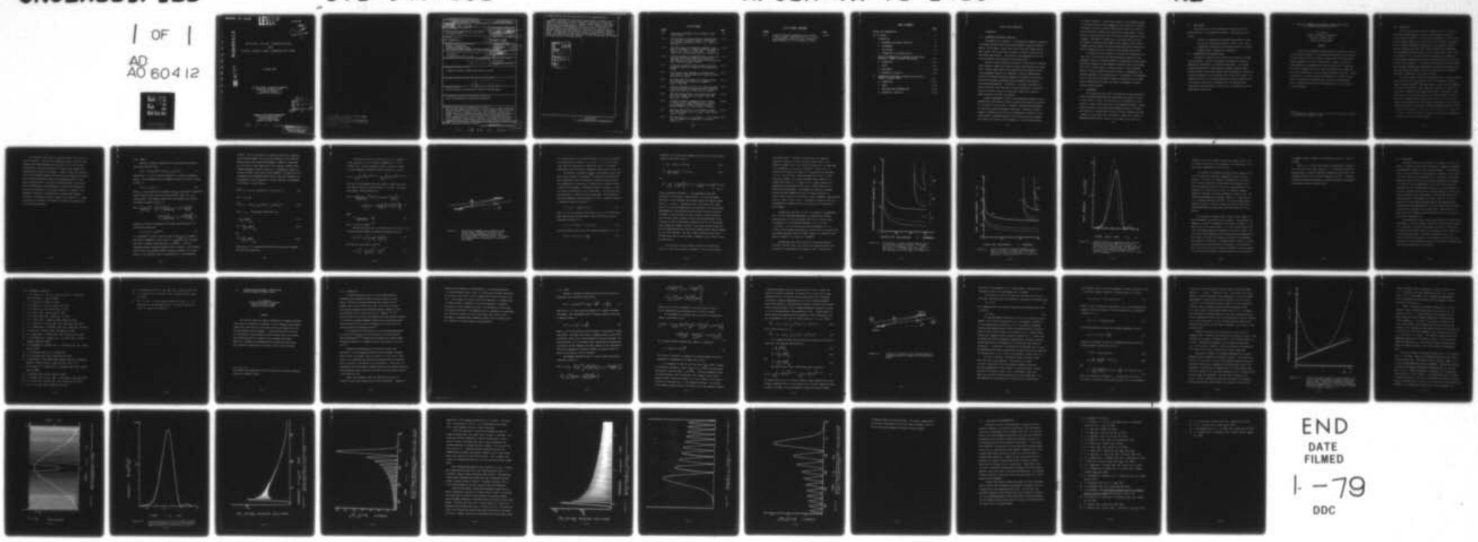

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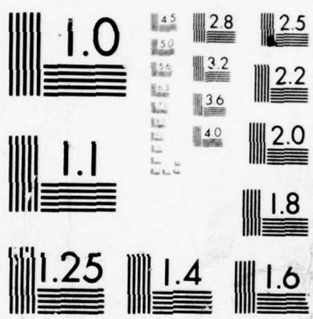

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OPTICAL PULSE COMPRESSION FOR LASER RADAR AND COMMUNICATIONS

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The classical theory for compression of optical pulse in linear, dispersive media has been generalized to include a carrier frequency which varies quadratically as well as linearly with time. The nonlinearly chirped pulse becomes temporally compressed upon propagating through a strongly dispersive medium. The group delay in the strongly dispersive propagation medium varies quadratically with carrier frequency. This theory has been applied to an experimental configuration of two gratings oriented to give strong dispersion of group delay. Objectives are to determine the influence of → next page

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strong dispersion upon the shape and width of the compressed pulse. micrometers

Results indicate that a grating-pair delay medium is feasible which is capable of compressing a linearly chirped 88 ns wide pulse at 10.6μ wavelength to a final width of 0.8 ns. However, strong dispersion does distort the envelope of the compressed pulse so that it is no longer strictly the Fourier transform of the envelope of the original pulse. Addition of a nonlinear chirp to the incident pulse, in the form of a quadratic change in carrier frequency with time, causes the output to be broken into a series of pulses. The sub-pulses in this series are much narrower than the incident pulse, and exhibit increasingly larger gain in peak intensity over the incident pulse toward the end of the pulse train.

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OPTICAL PULSE COMPRESSION

I. INTRODUCTION

I-A. BACKGROUND AND RESEARCH OBJECTIVES:

The purpose of this program is to investigate the temporal compression of frequency modulated laser pulses upon propagation through a dispersive medium. By this means, a broad laser pulse of high total energy but low peak intensity can be compressed to produce a relatively narrow pulse of much higher peak intensity. The envelope of the compressed pulse is the Fourier transform of the envelope of the original pulse.

The gain in peak intensity and reduction in width result in improved signal detection capabilities along with higher range resolution for laser radar. Greater range capability and compensation for dispersion effects in the propagation medium are of importance in optical communications. The Fourier transform relationship between the envelopes of the original pulse and the optimally compressed pulse has promising applications in signal waveform processing. In principle, the Fourier transform of an arbitrary waveform can be performed in analog fashion over a time period equal to the transit time of the light pulse through the dispersive medium, typically less than a microsecond.

A major disadvantage of optical pulse compression methods previously demonstrated is that they are limited to optical pulses which are typically 0.1 ns and less in initial width. Extremely large modulation bandwidths are required to compress nanosecond wide pulses. Extension of present methods to compress pulses which are one nanosecond or greater in initial width requires a propagation medium in which the group velocity changes very rapidly with optical carrier frequency, or in which the group delay

is strongly dispersive. The desirable feature of the propagation medium is that the group delay varies linearly but very rapidly with frequency. The problem is that for presently known dispersive media the linear dispersion required to compress one nanosecond or broader pulses is accompanied by significant higher order variations of group delay with carrier frequency. Additionally, methods such as self-phase modulation which are capable of producing very wideband linear frequency chirp (carrier frequency varying linearly with time) produce higher order variations of carrier frequency with time in addition to the desired linear chirp.

The objective of this program is to examine the influences of nonlinear chirp in the initial pulse and strong group dispersion in the propagation medium, and determine whether any combination of these factors might permit extension of these methods to compress laser pulses broader than 10 ns in initial width. Following classical, linear dispersion theory an original, closed-form analytic solution is developed for the frequency spectrum of the incident pulses. The envelope of the compressed pulse is then readily calculated by numerical evaluation of the inverse Fourier transform which includes the effects of propagation through a strongly dispersive medium.

I-B. ACHIEVEMENTS

It has been shown that a pair of gratings can be used to compress linearly chirped 88 nanosecond wide pulses from a CO_2 laser at 10.6μ by a ratio of 100:1 in width. The price paid for this capability is substantial in terms of the size and cost of the compression medium. However, these are economic issues rather than fundamental physical limitations. Upon addition of a second-order variation of carrier frequency with time in addition to the linear chirp, the output is broken into a series of compressed sub-pulses with consecutively increasing peak intensities.

I-C. PUBLICATIONS

One journal artical has been submitted for publication, and a second manuscript is in preparation at present. These articles are as follows:

1. "Analysis of Compression of Frequency Chirped Optical Pulses by a Strongly Dispersive Grating Pair," subm. J. Appl. Optics.
2. "Compression of Nonlinearly Chirped Optical Pulses in Strongly Dispersive Media," manuscript in preparation.

I-D. INTRODUCTION TO FOLLOWING SECTIONS

The technical discussion which follows in this report is divided into two sections. Section II represents the contents of an article submitted for publication, in slightly modified format. This section analyzes the extension of these methods to compress linearly chirped pulses which have initial widths greater than one nanosecond. Section III includes the possibility of a nonlinear chirp in the initial pulse. The contents of Section III are being expanded to include the possibility of a non-optimum linear chirp in combination with the nonlinear chirp. These results and the contents of Section III, in revised form, will be submitted for publication.

II. ANALYSIS OF COMPRESSION OF FREQUENCY CHIRPED OPTICAL PULSES BY A STRONGLY DISPERSIVE GRATING PAIR *

J. D. McMullen
Rockwell International Corporation,
Electronics Research Center
Anaheim, California 92803

ABSTRACT

An optical pulse compression medium formed by two parallel gratings can be strongly dispersive if the diffracted beam emerges at grazing angle from the surface of the first grating. Strong dispersion makes possible temporal compression of broad incident pulses. However, sufficiently strong dispersion is accompanied by a nonlinear variation of group delay with carrier frequency, which will cause the envelope of the compressed pulse to be distorted and limit the minimum attainable pulse width. Strong dispersion as a limiting influence upon compression of linearly chirped pulses is examined for the grating pair at several laser wavelengths. Specific examples are discussed for the compression of CO_2 laser pulses at 10.6μ wavelength.

* This research was supported by the Air Force Office of Scientific Research under Contract No. F49620-77-C-0109.

II-A. INTRODUCTION

An optical pulse having a carrier frequency which varies linearly with time may become temporally compressed upon propagating through a linear, dispersive medium.[1,2] A large pulse-width compression ratio requires a precise balance among initial pulse width, chirp or modulation bandwidth, and the frequency dependence of group delay in the dispersive medium.[1-4] Previous treatments have dealt with linearly chirped pulses propagating through moderately dispersive media in which the group delay varies linearly with frequency.[1,2] This classical approach has recently been extended to include frequency-dependent absorption and a group delay which varies quadratically with frequency.[3,4] It was shown for the latter case that the higher order frequency dependence may significantly influence the compressed pulse shape even though the quadratic variation of group delay with frequency is very small in comparison to the linear variation.[4]

At present, several methods have been demonstrated to compress chirped laser pulses less than one nanosecond in width by propagation through a linear, dispersive medium.[5-22] Many applications in laser radar and communications favor the extension of pulse compression techniques to larger initial pulse widths. Wigmore and Grischkowsky have recently converted a cw laser output into 0.5 ns wide pulses using a 56 ns frequency modulation period, equivalent to compressing 56 ns pulses by a ratio of 112:1.[23] In order to compress wide incident pulses, the propagation medium must be very strongly dispersive, because the difference in group delay between leading and trailing edges of the incident pulse must be as large as the initial pulse width. The quadratic dispersion parameter τ_2 previously defined must be comparable in magnitude to the initial pulse width.[3,4]

In principle, large values of τ_2 may be obtained using a pair of parallel gratings as the dispersive propagation medium. When the angle between the diffracted beam and the face normal of the first grating approaches 90°, the group dispersion of the grating pair becomes very large and varies rapidly with frequency. However, higher order contributions to the frequency dependent group delay may then become sufficiently large to limit the shape and width of the compressed pulse.[4] In this paper, the grating pair is analyzed to determine whether chirped laser pulses 10 nanoseconds to 100 nanoseconds in width can be compressed by using a pair of gratings in the strongly dispersive orientation. The fundamental limitation is determined by the higher order frequency variations of group delay which may accompany large values of τ_2. These limitations are discussed for the specific example of CO_2 laser pulses at 10.6 microns wavelength.

II-B. THEORY

Consider a linearly chirped optical pulse with Gaussian envelope which has an electric field

$$E(0,t) = E_o \exp(-2t^2/T^2) \cos\left(\omega_o t + \delta\omega_m t^2/(2T)\right) \qquad (1)$$

specified at $x = 0^+$ just inside the boundary of a dispersive propagation medium. The instantaneous carrier frequency, defined as the rate of change of phase, is

$$\omega(t) = \omega_o + \delta\omega_m t/T \quad , \qquad (2)$$

where ω_o is the average carrier frequency and $\delta\omega_m$ is the range of frequencies (rad/s) covered by the linear chirp during the full width T at 1/e of peak intensity. The frequency spectrum for the incident pulse, given by the Fourier transform of E(0,t), is

$$\tilde{E}(0,\omega) = \left(\frac{2\pi}{\delta\omega_o\ \delta\omega}\right)^{1/2} E_o \left\{ \exp\left[\frac{-2(\omega-\omega_o)^2}{\delta\omega_o(\delta\omega_o+i\delta\omega_m)} - i\,\frac{1}{2}\tan^{-1}\left(\frac{\delta\omega_m}{\delta\omega_o}\right)\right] + \exp\left[\frac{-2(\omega+\omega_o)^2}{\delta\omega_o(\delta\omega_o-i\delta\omega_m)} + i\,\frac{1}{2}\tan^{-1}\left(\frac{\delta\omega_m}{\delta\omega_o}\right)\right]\right\}, \qquad (3)$$

where $\delta\omega_o = 4/T$ is the 1/e bandwidth of the power spectrum $|\tilde{E}(0,\omega)|^2$ for the Gaussian envelope and

$$\delta\omega = [\ (\delta\omega_o)^2 + (\delta\omega_m)^2]^{1/2} \quad . \qquad (4)$$

It is convenient to refer to the first term of $\tilde{E}(0,\omega)$ which is peaked in amplitude at frequencies in the vicinity of $+\omega_o$ as $\tilde{E}^{(+)}(0,\omega)$ and the second term which is peaked in amplitude about $-\omega_o$ as $\tilde{E}^{(-)}(0,\omega)$. The pulse propagates through a dispersive medium in the +x direction. Each frequency group in $\tilde{E}(0,\omega)$ then becomes shifted in phase by an amount $\Phi(\omega) = \omega nx/c$, where n is the refractive index of the medium and x is the propagation

distance. The pulse envelope $E(x,t)$ becomes distorted and, depending upon the balance among T and $\delta\omega_m$ and the properties of the dispersive medium, may be either dispersion-broadened or temporally compressed. $\Phi(\omega)$ must vary at least quadratically with frequency, through either a frequency-dependent refractive index $n(\omega)$ or propagation distance $x(\omega)$, in order to obtain either type of pulse reshaping. The phase delay $\Phi(\omega)$ introduced by the propagation medium varies slowly with frequency over the bandwidth of the incident pulse, and may, therefore, be expressed in the form of a Taylor series as

$$\Phi^{(+)}(\omega) = \Phi_o + \tau_1(\omega-\omega_o) + \frac{1}{8}\tau_2^{\,2}(\omega-\omega_o)^2 + \frac{1}{3}\tau_3^{\,3}(\omega-\omega_o)^3 + \ldots \tag{5-a}$$

for $\omega \simeq +\omega_o$ and

$$\Phi^{(-)}(\omega) = \Phi_o - \tau_1(\omega+\omega_o) + \frac{1}{8}\tau_2^{\,2}(\omega+\omega_o)^2 - \frac{1}{3}\tau_3^{\,3}(\omega+\omega_o)^3 + \ldots \tag{5-b}$$

for $\omega \simeq -\omega_o$. The expansion coefficients are

$$\tau_1 = \frac{d}{d\omega}\left(\frac{\omega n x}{c}\right)\bigg|_{+\omega_o} , \tag{5-c}$$

$$\tau_2^{\,2} = 4\,\frac{d^2}{d\omega^2}\left(\frac{\omega n x}{c}\right)\bigg|_{+\omega_o} , \tag{5-d}$$

and

$$\tau_3^{\,3} = \frac{1}{2}\frac{d^3}{d\omega^3}\left(\frac{\omega n x}{c}\right)\bigg|_{+\omega_o} , \tag{5-e}$$

where $\Phi_o \equiv k_o x$. It is assumed that each Taylor series may be truncated after the third order term.

The electric field of the output pulse $E(x,t)$ is formed by linear superposition of the frequency components $\tilde{E}(0,\omega)$ in the incident field, with each component shifted in phase by an amount $\Phi(\omega)$ upon propagation through the dispersive medium, as follows:

$$E(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, \tilde{E}^{(+)}(0,\omega)e^{i\Phi^{(+)}(\omega)-i\omega t} + \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, \tilde{E}^{(-)}(0,\omega)e^{-i\Phi^{(-)}(\omega)-i\omega t}. \qquad (6)$$

The signs for the exponents above were chosen to insure that $|E(x,t)|$ diminishes to zero as $x \to +\infty$ when absorption is included. The output pulse shape is then evaluated to be

$$E(x,t) = \mathrm{Re}\left[\frac{2E_0}{\tau_3}\left(\frac{2\pi}{\delta\omega_0\delta\omega}\right)^{1/2} \exp\left\{ i\left[k_0x-\omega_0t-\frac{1}{2}\tan^{-1}\left(\frac{\delta\omega_m}{\delta\omega_0}\right)\right]\right\} \exp\left[\frac{a}{\tau_3{}^3}\left(\tau_1-t+\frac{2}{3}\frac{a^2}{\tau_3{}^3}\right)\right]\mathrm{Ai}\left(\frac{\tau_1-t+a^2/\tau_3{}^3}{\tau_3}\right)\right], \qquad (7)$$

where

$$a \equiv \frac{2}{\delta\omega_0(\delta\omega_0+i\delta\omega_m)} - i\,\frac{\tau_2{}^2}{8} \qquad (8)$$

and Ai is the Airy integral.[3, 24]

When the coefficient $\tau_3{}^3$ is sufficiently small that $\tau_3{}^3\delta\omega \ll \tau_2{}^2$, the pulse intensity reduces to a Gaussian envelope of the form

$$\left|E(x,t)\right|^2 = \left|E_0\right|^2 \frac{T}{T_x} \exp\left(-\frac{4(\tau_1-t)^2}{T_x{}^2}\right), \qquad (9)$$

with the final pulse width T_x given by

$$T_x = T\left\{\left[1+\frac{\delta\omega_m}{\delta\omega_0}\left(\frac{\tau_2}{T}\right)^2\right]^2 + \left(\frac{\tau_2}{T}\right)^4\right\}^{1/2} \qquad (10)$$

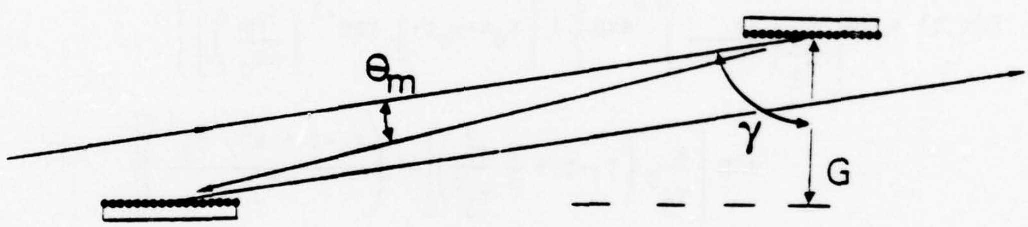

Figure II-1. Grating pair arrangement used to compress chirped optical pulses. The beam is incident at angle γ and the diffracted beam emerges at angle $\gamma-\theta_m(\omega)$ to the face normal of the first grating. The gratings are aligned parallel, with separation G along their face normals.

The minimum width which can then be obtained is $T_x = 4/\delta\omega_m$, corresponding to an optimized chirp range of $\delta\omega_m = -4T/\tau_2^2$. If τ_3 is sufficiently large, then the compressed pulse shape given by Eq. (7) may depart substantially from the optimally compressed, Gaussian pulse of Eq. (9).

The gratings illustrated in Figure 1 are oriented so that group delay varies strongly with frequency.[5-7] The gratings are aligned with their faces parallel and are separated a distance G along the direction perpendicular to both faces. An optical pulse incident at angle γ upon the first grating is diffracted in order m at angle $\theta_m(\omega)$ from the direction antiparallel to the incident beam. Light emerging from the grating pair is parallel to the incident beam. Each frequency in the pulse propagates a unique distance $x(\omega)$ and is, therefore, delayed in phase by an angle $\Phi(\omega)$ given by[7]

$$\Phi(\omega) = \frac{\omega}{c}\, x(\omega) - \frac{2\pi G}{d} \tan\left(\gamma - \theta_m(\omega)\right) , \tag{11}$$

where d is the line spacing of the grating, $\gamma - \theta_m(\omega)$ is the diffraction angle measured from the face normal of the first grating, and the propagation distance $x(\omega)$ is

$$x(\omega) = G\left(1 + \cos\theta_m(\omega)\right) / \cos\left(\gamma - \theta_m(\omega)\right) \quad . \tag{12}$$

The grating equation which gives the frequency dependence of $\theta_m(\omega)$ is

$$\sin(\gamma) + \sin\left(\gamma - \theta_m(\omega)\right) = m\, \frac{2\pi c}{\omega d} \quad . \tag{13}$$

Equations (11)-(13) and their frequency derivatives yield the following expansion coefficients for $\Phi(\omega)$:

$$\tau_1 = \frac{G}{c}\,(1 + \cos\theta_m) \,/\, \cos(\gamma-\theta_m) \quad , \tag{14-a}$$

$$\tau_2^{\,2} = \frac{-4G}{c\omega_o}\left(\frac{m\,2\pi c}{\omega_o d}\right)^2 \cos^{-3}(\gamma-\theta_m) \quad , \tag{14-b}$$

and

$$\tau_3^{\,3} = \frac{3G}{2c\omega_o^{\,2}}\left(\frac{m\,2\pi c}{\omega_o d}\right)^2\left[\cos^2(\gamma-\theta_m) + \left(\frac{m\,2\pi c}{\omega_o d}\right)\sin(\gamma-\theta_m)\right]\cos^{-5}(\gamma-\theta_m), \tag{14-c}$$

with θ_m evaluated at frequency ω_o. The magnitude of the factor $(m\,2\pi c/\omega_o d)$ is restricted by the grating Eq. (13) to values between zero and 2, so that estimates can be easily made of the angular dependence of τ_1, τ_2 and τ_3. Examination of Eq. (14-b) reveals that larger values of τ_2 for pulse compression can be obtained if (a) the diffracted beam emerges at a grazing angle from the surface of the first grating ($\gamma-\theta_m \simeq 90°$), (b) the gratings are used in nearly a Littrow configuration with small angle θ_m between incident and diffracted beams so that $m\,2\pi c/\omega_o d$ is close to the maximum allowable value of 2, (c) the perpendicular separation G between gratings is very large, perhaps through the use of a folded optical path, and (d) the average optical carrier frequency ω_o is as low as possible. The last of these conditions favors use of infrared lasers, such as the CO_2 laser at frequency $\omega_o/2\pi c = 943\ \text{cm}^{-1}$, with the grating pair.

The grating pair exhibits negative dispersion of group delay ($\tau_2^{\,2} < 0$), so that lower frequency components in the pulse are delayed

the greatest amount. Therefore, the grating pair can generate a negatively chirped pulse from an unchirped incident pulse, or can temporally compress a positively chirped incident pulse. The magnitudes of the parameters τ_2 and τ_3 are illustrated in Figures 2 and 3 as functions of the angle of incidence γ, for average carrier wavelengths of 10.6μ, 1.06μ and 0.48μ. These wavelengths are typical of pulsed laser radar systems. The shorter wavelengths remain potentially of interest even though $|\tau_2|$ is largest at 10.6μ, because greater chirp bandwidths can be attained by electro-optic frequency modulation at shorter optical wavelengths. The perpendicular separation G between gratings has been standardized at 100 m for these calculations in order to obtain values of τ_2 sufficiently large to compress a nanosecond-wide pulse by a factor of 100 at 10.6μ. Three possible values for $m2\pi c/(\omega_o d)$, corresponding to different grating line spacings, are illustrated for each frequency.

Figures 2 and 3 show that $|\tau_2|$ and $|\tau_3|$ diverge as $\gamma-\theta_m$ approaches 90°, or as the diffracted beam emerges at a grazing angle from the grating surface. For 10.6μ wavelength, $m2\pi c/(\omega_o d)=1.9$, and an angle of incidence $\gamma=71°$ the dispersion parameters are $|\tau_2|=1$ ns and $|\tau_3|=0.036$ ns. A 10 ns wide pulse can then be compressed to 0.1 ns in width if the incident pulse contains a positive chirp of $\delta\omega_m/2\pi=6.37$ GHz. The incident beam must be well collimated, because $|\tau_2|$ changes with the angle of incidence γ at a rate of 0.1 ns/degree.

The compressed pulse has an essentially undistorted Gaussian envelope if the cubic term in Eqs. (5-a,b) has a magnitude smaller than one percent of the magnitude of the quadratic term.[3] Therefore, the

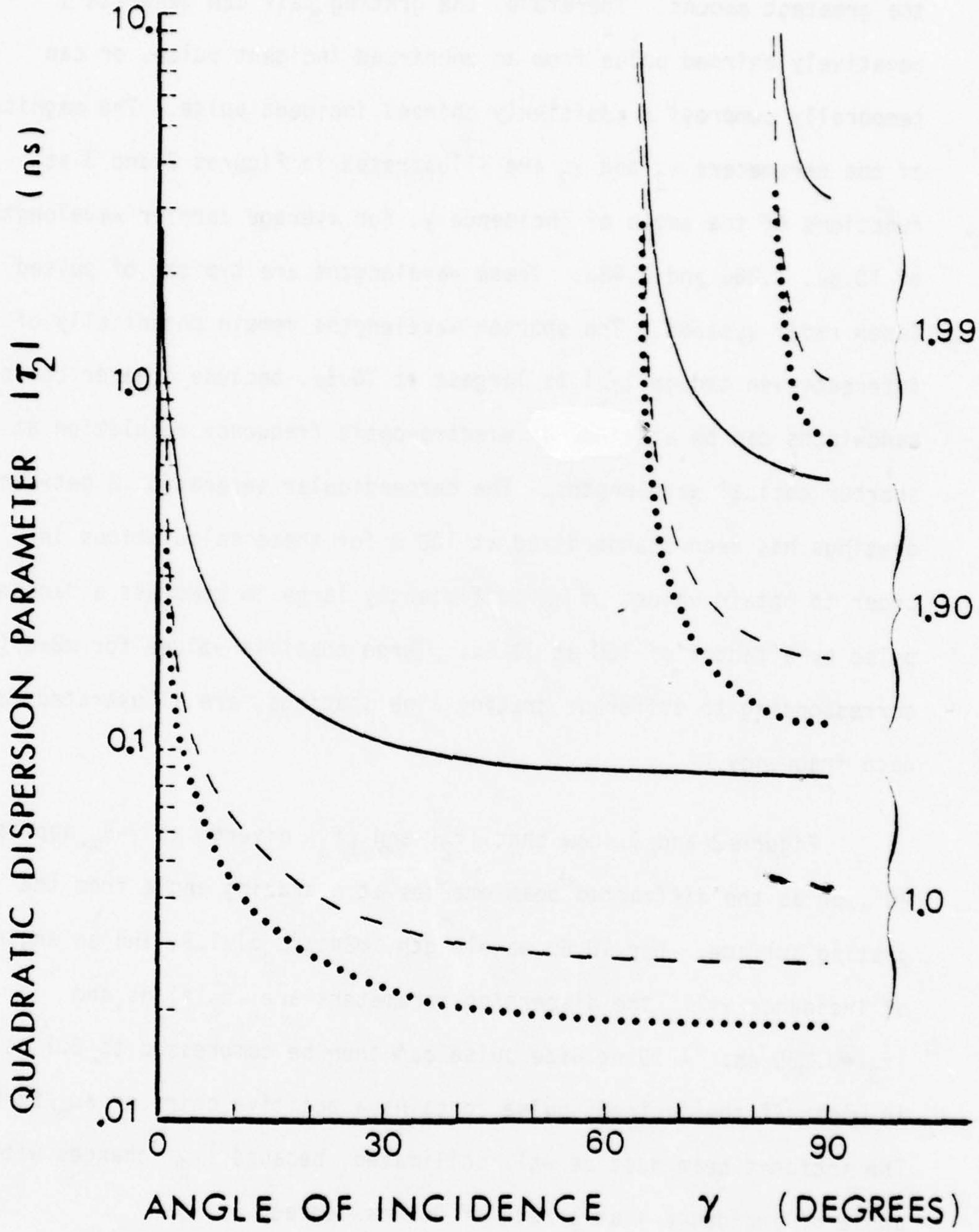

Figure II-2. Relative delay $|\tau_2|$ among different frequency groups as a function of angle of incidence γ for average optical carrier wavelengths 10.6μ (——), 1.06μ (----) and 0.48μ (····). The three cases illustrated for each wavelength are for $m2\pi c/(\omega_0 d)$=1.0, 1.90 and 1.99.

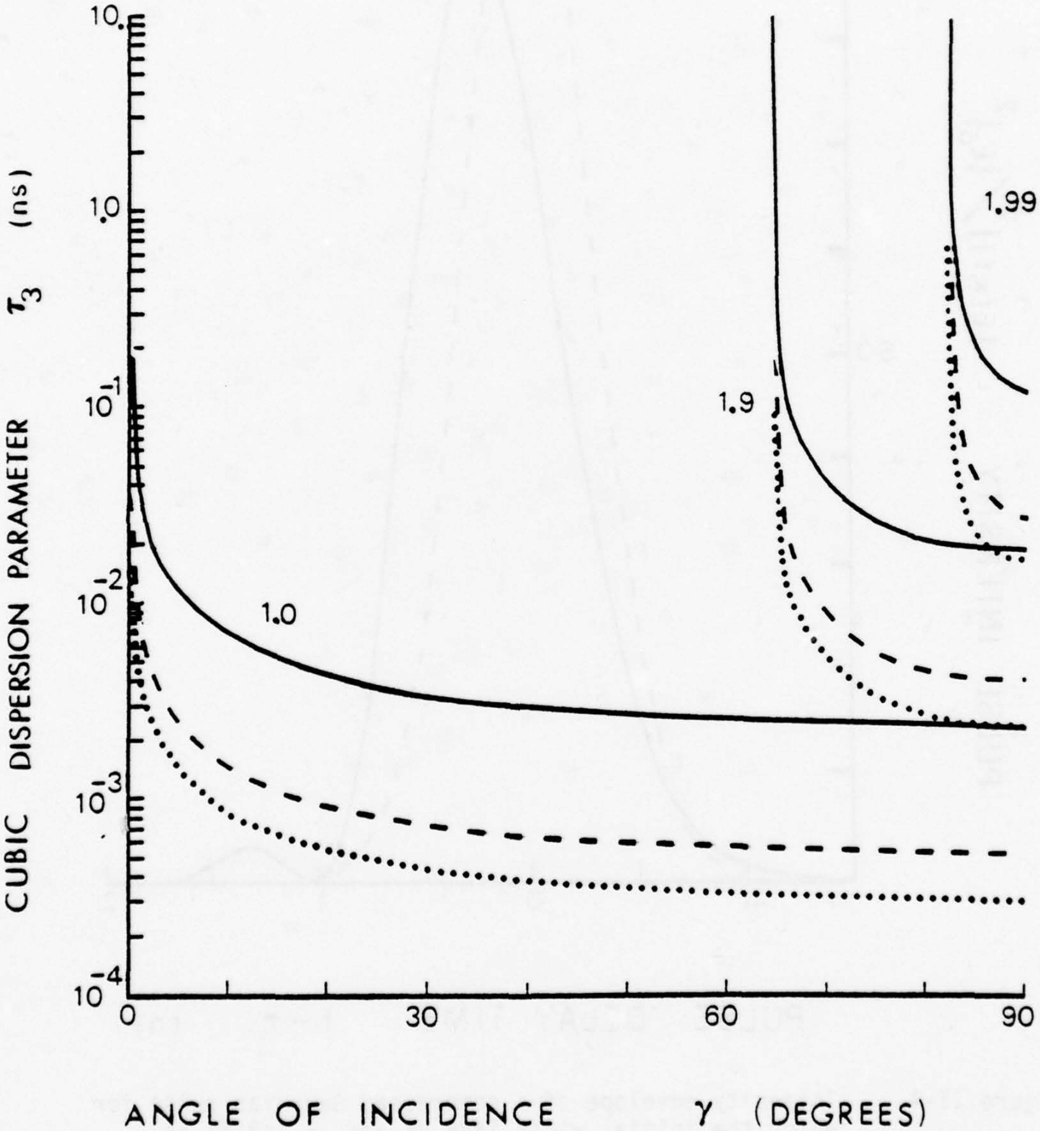

Figure II-3. Variation of the cubic dispersion parameter τ_3 with angle of incidence γ at average carrier wavelengths of 10.6μ (——), 1.06μ (----) and 0.48μ (····), using grating pairs for which $m2\pi c/(\omega_o d)$=1.0, 1.90 and 1.99.

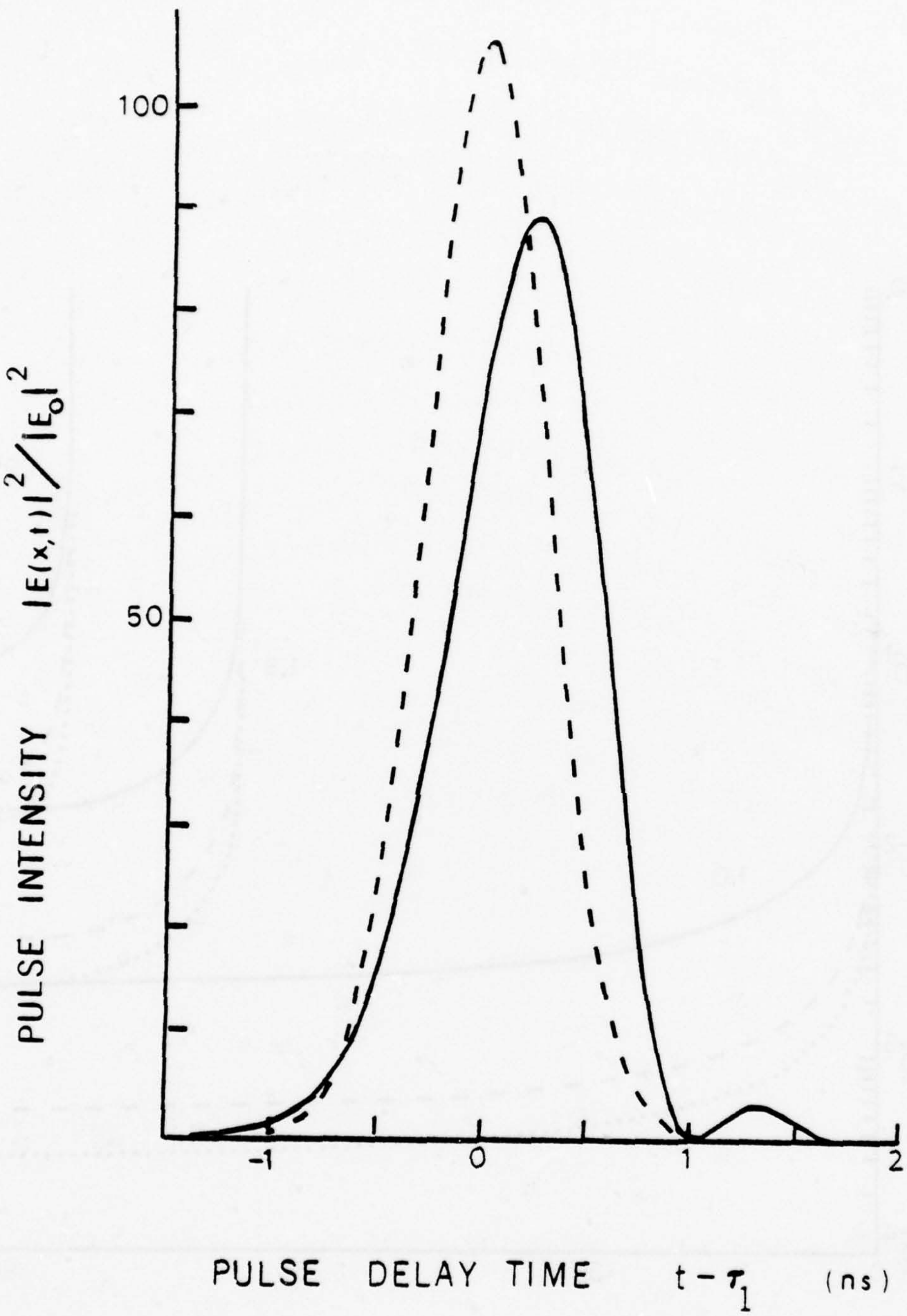

Figure II-4. Intensity envelope of a compressed Gaussian pulse for which the initial width T=88 ns and $|\tau_2|$=8.54 ns, obtained using a grating pair for which $m2\pi c/(\omega_o d)$=1.90 and a linear chirp $\delta\omega_m/2\pi$=769 MHz. The idealized case τ_3=0 (----) gives a Gaussian envelope. The case τ_3=0.384 ns (——) includes nonlinear variation of group delay with frequency.

bandwidth $\delta\omega/2\pi$ of the incident pulse must not exceed 12.8 GHz. For an incident pulse having a chirp bandwidth of this magnitude, a 20 ns pulse may be compressed to a width of 48 ps.

The minimum chirp bandwidth which may be useful occurs for the case $T=T_x$ where the incident and "compressed" pulses are of equal duration. This case is of interest if the objective is to form an optical Fourier transform of the envelope of the incident pulse.[1,3] The chirp bandwidth required for this case is $\delta\omega_m/2\pi=637$ MHz. Chirp bandwidths this large are very difficult to achieve by frequency modulation techniques for optical wavelengths as large as 10.6μ. The bandwidth requirements may be reduced, however, if a larger value of $|\tau_2|$ is obtained from the grating pair. This can be accomplished by reducing the angle of incidence γ toward the value for which the diffracted beam emerges at grazing angle from the surface of the first grating.

For an angle of incidence $\gamma=64.5°$, Figures 2 and 3 reveal dispersion parameters $|\tau_2|=8.54$ ns and $|\tau_3|=0.384$ ns. The magnitude of τ_2 changes at a rate of 8.3 ns/degree with the angle of incidence γ. The penalty paid for the larger value of τ_2 is that strong dispersion now limits the maximum useable chirp bandwidth to a value of 769 MHz. For this value of chirp bandwidth an 88 ns wide pulse may be compressed to a final width of 0.83 ns, with the Gaussian envelope of the compressed pulse essentially preserved. The dashed curve of Figure 4 illustrates the Gaussian pulse shape which would be obtained if τ_3 was negligibly small, while the solid curve includes the influence of strong dispersion. For the latter case the peak is further delayed and the pulse envelope

is asymmetric about that peak, with interference structure in the trailing edge.

When $T_x = T = 8.54$ ns, the minimum chirp bandwidth of interest is only 74.5 MHz for this larger value of $|\tau_2|$. The nonlinear group delay characterized by τ_3 exhibits negligible influence over the shape of the compressed pulse for an initial chirp bandwidth this small. Therefore, the Fourier transform relationship is preserved between the envelopes of the original pulse and the output pulse.

II-C. CONCLUSIONS

Nonlinear variation of group delay with frequency can distort the shape of a linearly chirped optical pulse which is temporally compressed upon propagation through a strongly dispersive medium. This distortion serves to limit application of the grating-pair method to optical pulses of initial widths smaller than about 100 ns. For example, it has been shown that an 88 ns wide CO_2 laser pulse with average carrier frequency $\omega_0/2\pi c=943\ cm^{-1}$ can be compressed by a ratio 106:1 using gratings for which $m2\pi c/\omega_0 d=1.9$. Although larger values of $|\tau_2|$ may be obtained, in principle, by decreasing the angle of incidence toward the limiting value of 64.16° allowed by the grating equation, the nonlinear contribution characterized by τ_3 then becomes sufficiently large to influence the shape and limit the minimum attainable width of the compressed pulse. The envelope of the compressed pulse is then no longer simply the Fourier transform of the envelope of the incident pulse. Larger values of $|\tau_2|$ may be obtained to compress broader chirped pulses only if $m2\pi c/(\omega_0 d)$ is made closer in magnitude to 2.

The grating pair produces larger group dispersion at longer optical wavelengths, so that this method is favored for use with the CO_2 laser at 10.6μ wavelength. The primary experimental difficulty is production of the required chirp bandwidth over the time duration of the incident pulse at these long wavelengths. Additional experimental factors which should be carefully considered are the collimation required of the incident beam and the wide dimensions required for the grating apertures along the direction perpendicular to the grooves.

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24. Eq. (6) of Ref. (3) should contain an additional factor τ_3 in the denominator of the amplitude of $E(x,t)$. The right side of Eq. (4) in Ref. (3) should be divided by 2π.

III. COMPRESSION OF NONLINEARLY CHIRPED OPTICAL PULSES IN STRONGLY DISPERSIVE MEDIA*

J. D. McMullen
Rockwell International Corporation,
Electronics Research Center
Anaheim, California 92803

ABSTRACT

The classical theory for temporal compression of frequency modulated light pulses is extended to include both a carrier frequency which changes quadratically with time and a group delay which varies quadratically with the instantaneous carrier frequency. A closed-form solution is derived for the spectrum of the nonlinearly chirped incident pulse, which greatly facilitates calculation by computer of the compressed pulse shape. Addition of a quadratic time dependence to an already optimized linear chirp causes the compressed pulse to break up into a series of pulses.

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III-A. INTRODUCTION

A frequency modulated optical pulse may become temporally compressed upon propagating through a suitably dispersive, linear medium. Previously considered approaches use an optical carrier frequency which varies linearly with time. This linearly chirped pulse becomes compressed upon propagating through a medium for which the group delay varies linearly with carrier frequency.[1,2] The classical theory for chirped pulse compression has been recently extended to include both frequency dependent absorption and a group delay which varies quadratically with carrier frequency, such as may occur for a strongly dispersive propagation medium.[3-5]

The compression of chirped optical pulses has been demonstrated by a variety of techniques for pulses having initial widths smaller than one nanosecond.[4-23] Wigmore and Grischkowsky have recently demonstrated the feasibility of compressing 56 ns wide pulses by a ratio of 112:1.[24]

Substantial advantages would be realized in several laser applications if pulse compression methods could be extended to broader optical pulses; for example, 100 ns or greater in initial width. Pulse widths of this magnitude are produced by Q-switched lasers at repetition rates of typically a few hundred to a few thousand Hz. These lasers, in combination with wide bandwidth frequency modulation and strongly dispersive pulse compression media, potentially may provide high peak power pulses at a high repetition rate.

Linear chirp bandwidths wider than 200 MHz are difficult to achieve by electro-optic modulation at infrared wavelengths. Attempts to

extend the chirp bandwidth by these methods or by self-phase modulation in a nonlinear medium usually result in a significant nonlinear contribution to the chirp, or an optical carrier frequency which varies nonlinearly with time. In this paper the classical theory for compression of chirped pulses in dispersive media is extended to include the presence of a quadratic variation of carrier frequency with time. The influence of this nonlinear chirp upon the envelope of the compressed pulse is examined for an experimentally realizable situation in which an 88 ns wide laser pulse at 10.6 μ wavelength is compressed upon propagating through a pair of gratings which are oriented in a strongly dispersive configuration.[5]

III-B. THEORY

Consider a nonlinearly chirped optical pulse with Gaussian envelope which has an electric field given by

$$E(0,t) = E_o \exp(-2t^2/T^2) \cos\left(\omega_o t + \frac{\delta\omega_m t^2}{2T} + \delta\omega_c \frac{4t^3}{3T^2}\right) \qquad (1)$$

specified at $x = 0^+$ just inside the boundary of a dispersive propagation medium. The instantaneous carrier frequency, defined as the rate of change of phase, is

$$\omega(t) = \omega_o + \delta\omega_m \frac{t}{T} + \delta\omega_c \frac{4t^2}{T^2} \quad , \qquad (2)$$

where ω_o is the fixed optical carrier frequency in the absence of higher order terms. The linear chirp covers a frequency range $\delta\omega_m$ (rad/s) during the full width at 1/e of peak intensity of the pulse envelope. The last term of Eq. (2) is a nonlinear chirp, consisting of a second-order variation of carrier frequency with time. Because of this contribution, the carrier frequency is raised by an amount $\delta\omega_c$ (rad/s) at both 1/e points on the Gaussian intensity envelope.

The frequency spectrum of the input pulse, given by the Fourier transform of E(0,t), is

$$\tilde{E}(0,\omega) = E_o \frac{\pi}{\delta\omega_o} \left(\frac{4\delta\omega_o}{\delta\omega_c}\right)^{1/3} \left\{ \exp\left(\frac{(\delta\omega_o + i\delta\omega_m)}{2\delta\omega_o\,\delta\omega_c}\left[-(\omega-\omega_o) + \frac{(\delta\omega_o + i\delta\omega_m)^2}{24\delta\omega_c}\right]\right)\right.$$

$$Ai\left(-\frac{(\omega-\omega_o)}{(\delta\omega_c\,\delta\omega_o^{\,2}/4)^{1/3}} + \frac{(\delta\omega_o + i\delta\omega_m)^2}{(4^5\delta\omega_c^{\,4}\delta\omega_o^{\,2})^{1/3}}\right) +$$

$$\exp\left(\frac{\left(\delta\omega_o - i\delta\omega_m\right)}{2\delta\omega_o\delta\omega_c}\left[(\omega+\omega_o) + \frac{\left(\delta\omega_o - i\delta\omega_m\right)^2}{24\delta\omega_c}\right]\right) \cdot$$

$$\left. Ai\left(\frac{(\omega+\omega_o)}{\left(\delta\omega_c\ \delta\omega_o{}^2/4\right)^{1/3}} + \frac{\left(\delta\omega_o - i\delta\omega_m\right)^2}{\left(4^5\delta\omega_c{}^4\ \delta\omega_o{}^2\right)^{1/3}}\right)\right\} \tag{3}$$

where $\delta\omega_o \equiv 4/T$ is the 1/e bandwidth of the power spectrum $|\tilde{E}(0,\omega)|^2$ of the (unmodulated) pulse envelope and Ai is the Airy integral.

In the absence of a nonlinear chirp, the Fourier transform $\tilde{E}(0,\omega)$ reduces to the familiar form

$$\lim_{\delta\omega_c = 0} \tilde{E}(0,\omega) = E_o\left(\frac{2\pi}{\delta\omega_o\delta\omega}\right)^{1/2}\left\{\exp\left[\frac{-2(\omega-\omega_o)^2}{(\delta\omega)^2} + \frac{i2\delta\omega_m(\omega-\omega_o)^2}{\delta\omega_o(\delta\omega)^2} + i\,\frac{1}{2}\tan^{-1}\left(\frac{\delta\omega_m}{\delta\omega_o}\right)\right]\right.$$

$$\left. + \exp\left[\frac{-2(\omega+\omega_o)^2}{(\delta\omega)^2} - \frac{i2\delta\omega_m(\omega+\omega_o)^2}{\delta\omega_o(\delta\omega)^2} - i\,\frac{1}{2}\tan^{-1}\left(\frac{\delta\omega_m}{\delta\omega_o}\right)\right]\right\}, \tag{4}$$

for a linearly chirped, Gaussian pulse, where $\delta\omega$ is defined as

$$\delta\omega = \left[(\delta\omega_o)^2 + (\delta\omega_m)^2\right]^{1/2} . \tag{5}$$

This limit will be useful for comparison to the more general case in the computations of compressed pulse shapes which follow.

It is convenient to refer to the first term of $\tilde{E}(0,\omega)$ which is peaked in amplitude at frequencies in the vicinity of $+\omega_o$ as $\tilde{E}^{(+)}(0,\omega)$ and the second term peaked in amplitude about $-\omega_o$ as $\tilde{E}^{(-)}(0,\omega)$. The pulse propagates through a dispersive medium in the $+\hat{x}$ direction. Each frequency group in $\tilde{E}(0,\omega)$ becomes shifted in phase by an amount $\Phi(\omega) = \omega nx/c$, where n is the index of refraction of the medium and x is the propagation distance. The relative shifts amoung frequency groups of the original pulse cause temporal reshaping

of the pulse envelope, giving a pulse at position x which is either compressed or dispersion broadened. The phase shift $\Phi(\omega)$ must vary at least quadratically with ω in order to realize either type of pulse reshaping. The quadratic frequency dependence may arise from a frequency-dependent index of refraction $n(\omega)$ associated with electronic transitions and infrared active vibrational modes or from a frequency-dependent propagation distance $x(\omega)$ as for the grating-pair pulse compression filter. The phase delay $\Phi(\omega)$ varies slowly with frequency over the bandwidth of the incident pulse, and may be expressed in the form of a Taylor series for $\omega \simeq +\omega_o$ as follows

$$\overset{(+)}{\Phi}(\omega) = \Phi_o + \tau_1(\omega-\omega_o) + \frac{1}{8}\tau_2{}^2(\omega-\omega_o)^2 + \frac{1}{3}\tau_3{}^3(\omega-\omega_o)^3 + \ldots, \tag{6-a}$$

while in the vicinity of $\omega \simeq -\omega_o$

$$\overset{(-)}{\Phi}(\omega) = \Phi_o - \tau_1(\omega+\omega_o) + \frac{1}{8}\tau_2{}^2(\omega+\omega_o)^2 - \frac{1}{3}\tau_3{}^3(\omega+\omega_o)^3 + \ldots \tag{6-b}$$

It is assumed that each Taylor series may be truncated after the cubic order term. The expansion coefficients are

$$\tau_1 = \frac{d}{d\omega}\left(\frac{\omega n x}{c}\right)\Bigg|_{\omega_o}, \tag{6-c}$$

$$\tau_2{}^2 = 4\frac{d^2}{d\omega^2}\left(\frac{\omega n x}{c}\right)\Bigg|_{\omega_o}, \tag{6-d}$$

and

$$\tau_3{}^3 = \frac{1}{2}\frac{d^3}{d\omega^3}\left(\frac{\omega n x}{c}\right)\Bigg|_{\omega_o}, \tag{6-e}$$

The electric field $E(x,t)$ of the output pulse is given by

$$E(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \overset{(+)}{\tilde{E}}(0,\omega)e^{i\overset{(+)}{\Phi}(\omega) - i\omega t} + \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \overset{(-)}{\tilde{E}}(0,\omega)e^{-i\overset{(-)}{\Phi}(\omega) - i\omega t} \tag{7}$$

The output field $E(x,t)$ is, in effect, formed by linear superposition of the frequency components $\tilde{E}(0,\omega)$ in the incident field, with each component shifted in phase by an amount $\Phi(\omega)$ upon propagation through the dispersive medium.

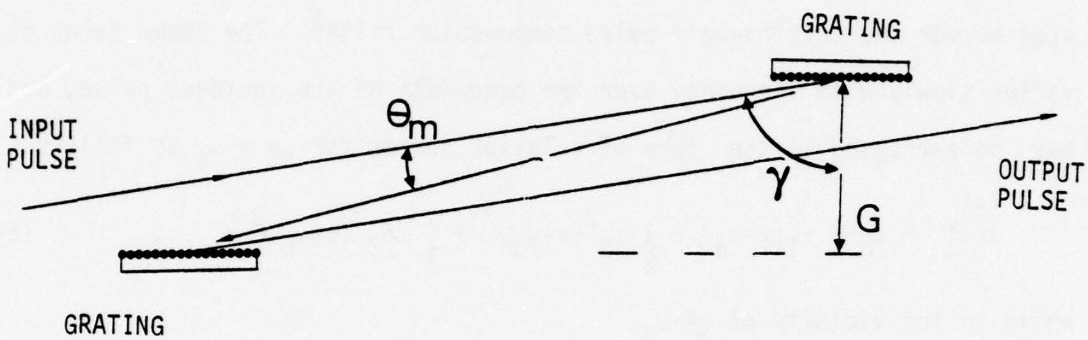

Figure III-1. Grating pair oriented to form a strongly dispersive propagation medium for compression of chirped optical pulses.

The signs for the exponents in Eq. (7) were chosen to insure that $|E(x,t)|$ diminishes to zero as $x \to +\infty$ when absorption is included.

The solution for $|E(x,t)|^2$ in the limit that both $\delta\omega_c=0$ and $\tau_3=0$ is well known. When the linear chirp bandwidth is adjusted to the optimum value

$$\delta\omega_m = 4T/\tau_2^2 \tag{8}$$

the compressed pulse has a Gaussian shape with the minimum possible width T_x given by

$$T_x = 4/\delta\omega_m \quad . \tag{9}$$

For the more general case where $\delta\omega_c$ and τ_3 are not negligibly small, however, $E(x,t)$ must be evaluated from Eq. (7) by computer, using $\tilde{E}(0,\omega)$ for the incident pulse as given by Eq. (3). The significant advantage offered by the present analysis is that a closed-form analytic solution is given for $\tilde{E}(0,\omega)$ which includes the nonlinear chirp $\delta\omega_c$, thereby greatly reducing the computation effort required to obtain $E(x,t)$. In order that the present calculations represent an experimentally realizable situation, consider specifically the compression of a chirped CO_2 laser pulse by a grating pair. In particular, the influence of the nonlinear chirp $\delta\omega_c$ upon the compressed pulse shape is examined for the case in which group dispersion of the grating pair is very strong, as manifested by a nonzero value of τ_3.

The pair of gratings illustrated in Figure 1 produces a group delay which varies with frequency. The gratings are aligned with their faces parallel and are separated a distance G along the direction perpendicular to both surfaces. An optical pulse is incident upon the first grating at angle γ and is diffracted in order m at angle $\theta_m(\omega)$ from the direction antiparallel to the incident beam. The beam emerging from the grating pair is parallel to the incident beam.

Each frequency group in the pulse propagates a different distance $x(\omega)$ and is, therefore, delayed in phase by a different angle $\Phi(\omega)$, given by

$$\Phi(\omega) = \frac{\omega}{c}\ x(\omega) - 2\pi \frac{G}{d}\ \tan\left(\gamma - \theta_m(\omega)\right) \quad , \tag{10}$$

where d is the line spacing of the grating, $\gamma-\theta_m$ is the diffraction angle measured from the face normal of the first grating, and the propagation distance $x(\omega)$ is

$$x(\omega) = G\ (1 + \cos\theta_m)/\cos(\gamma-\theta_m)\ . \tag{11}$$

The grating equation which gives the frequency dependence of $\theta_m(\omega)$ is

$$\sin(\gamma) + \sin(\gamma-\theta_m) = m\ \frac{2\pi c}{\omega d} \quad . \tag{12}$$

Equations (10) through (12) and their frequency derivatives yield the following expansion coefficients for $\Phi(\omega)$:

$$\tau_1 = \frac{G}{c}\ (1 + \cos\theta_m)/\cos(\gamma-\theta_m)\ , \tag{13-a}$$

$$\tau_2{}^2 = \frac{-4G}{c\omega_o} \left(\frac{m\ 2\pi c}{\omega_o d}\right)^2 \cos^{-3}(\gamma-\theta_m)\ , \tag{13-b}$$

and

$$\tau_3{}^3 = \frac{3G}{2c\omega_o{}^2} \left(\frac{m\ 2\pi c}{\omega_o d}\right)^2 \cos^2(\gamma-\theta_m) + \frac{m\ 2\pi c}{\omega_o d}\ \sin(\gamma-\theta_m)\cos^{-5}(\gamma-\theta_m), \tag{13-c}$$

where θ_m is evaluated at frequency ω_o. The magnitude of the factor $(m\ 2\pi c/\omega_o d)$ is restricted by the grating equation (12) to values between

zero and 2, so that estimates can be easily made of the angular dependences of τ_1, τ_2 and τ_3. Examination of Eq. (13-b) reveals that larger values of τ_2 can be obtained for pulse compression if (a) the diffracted beam emerges at a grazing angle from the surface of the first grating ($\gamma-\theta_m \simeq 90°$), (b) the gratings are used in nearly a Littrow configuration with small angle θ_m between incident and diffracted beams so that $m\ 2\pi c/\omega_o d$ is close to the maximum allowable value of 2, (c) the perpendicular separation G between gratings is very large, perhaps through the use of a folded optical path, and (d) the average optical carrier frequency ω_o is as low as possible. The last of these conditions favors use of infrared lasers such as the CO_2 laser at frequency $\omega_o/2\pi c = 943\ cm^{-1}$ with the grating pair.

Consider specifically a CO_2 laser pulse at $\omega_o/2\pi c = 943\ cm^{-1}$ frequency which is incident upon a grating pair for which $m2\pi c/(\omega_o d) = 1.90$ and G=100 m at an angle of incidence γ=64.5°. The diffraction angle $\gamma-\theta_m$ is 85.9°. The dispersion parameters for this dispersive medium are $|\tau_2|$ = 8.54 ns and τ_3 = 0.384 ns. An 88 ns wide pulse can be compressed to a width of 0.8 ns using a linear chirp of $\delta\omega_m/2\pi$ = 768.2 MHz.

Three cases of interest are presented corresponding to the time dependent carrier frequencies illustrated in Figure 2. The simplest case is that of a linear chirp, with the chirp bandwidth adjusted to the optimum value of 768.2 MHz for compression by the grating pair above. The second case, illustrated by the dashed curve, is for the previous situation with a quadratic chirp of moderate bandwidth $\delta\omega_c/2\pi$ = 159 MHz, smaller than the linear chirp bandwidth. The third case illustrated is for $\delta\omega_c/2\pi$ = 1.82 GHz, considerably larger than the linear chirp bandwidth. For the last two cases, the influence of the quadratic chirp upon the

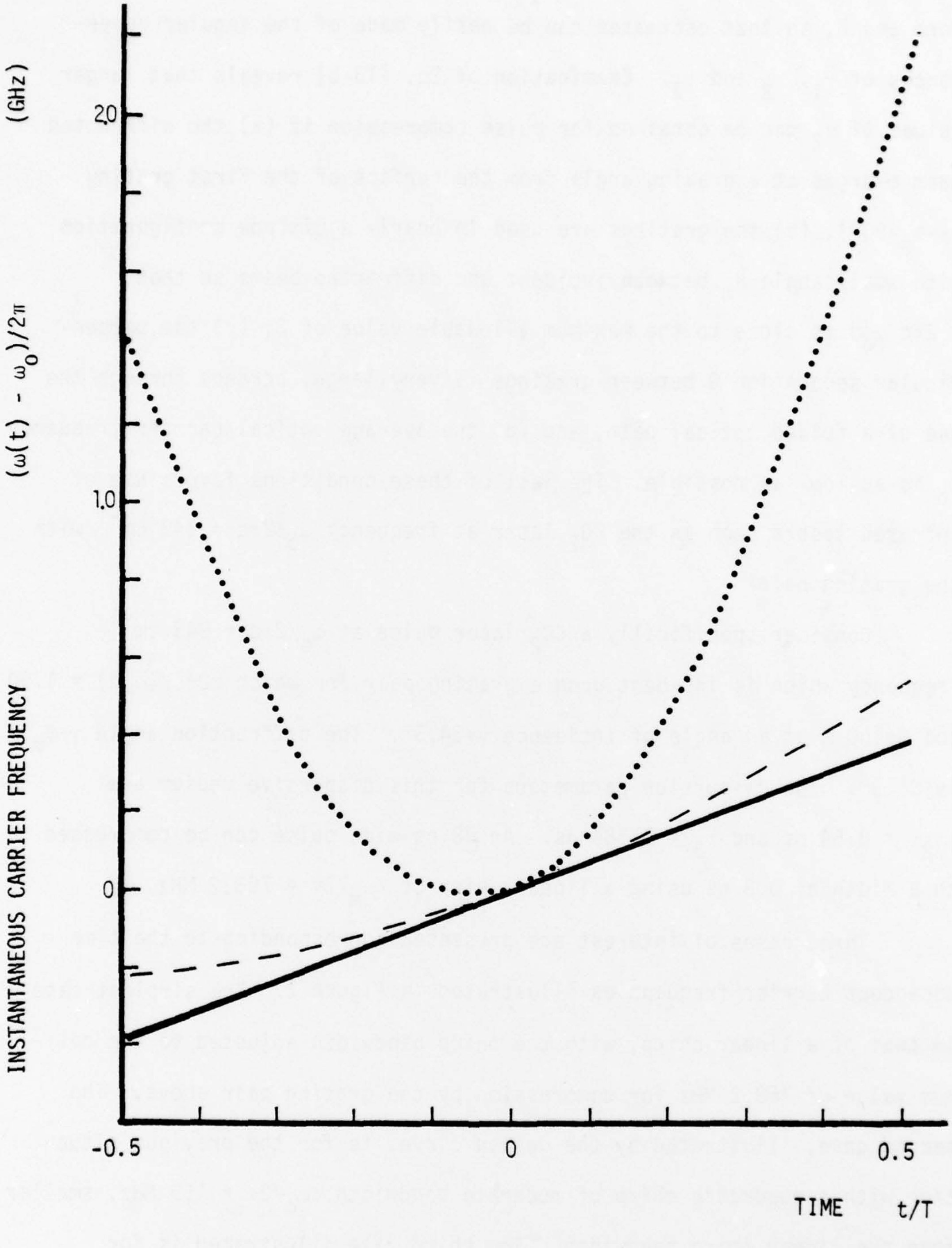

Figure III-2. Time-varying carrier frequency for optimum linear chirp $\delta\omega_m/2\pi$ = 768.2 MHz required to compress an 88 ns pulse to 0.8 ns width. Three cases illustrated include the addition of a quadratically varying carrier frequency of additional bandwidth $\delta\omega_c/2\pi$ = 0 (——), $\delta\omega_c/2\pi$ = 159 MHz (----), and $\delta\omega_c/2\pi$ = 1.82 GHz (····).

shape of the output pulse is determined by numerical evaluation of Eq. (7) using $\tilde{E}(0,\omega)$ for the incident pulse from Eq. (3). As indicated in Eq. (2), the quadratic chirp increases the instantaneous carrier frequency at both the leading and trailing edges of the incident pulse by an amount $\delta\omega_c$.

Figure 3 illustrates the frequency spectrum $|\tilde{E}(0,\omega)|$ of the Gaussian, linearly chirped incident pulse E(0,t) from Eq. (4). The envelope of $|\tilde{E}(0,\omega)|$ is likewise Gaussian, and the phase of $\tilde{E}(0,\omega)$ varies quadratically with frequency symmetrically about the average carrier frequency ω_o. The width of the spectrum is essentially the linear chirp bandwidth, which exceeds that of the pulse envelope by a factor of 106. Consider the above example, where this 88 ns pulse with a linear chirp of 768.2 MHz propagates through a grating pair for which $|\tau_2|$ = 8.54 ns and τ_3 = 0.384 ns. The intensity envelope of the 0.8 ns compressed output pulse is illustrated in Figure 4. Note that strong dispersion, specified by τ_3, limits the peak intensity attainable and causes interference structure in the trailing edge of the compressed pulse.

Figure 5 illustrates the frequency spectrum $|\tilde{E}(0,\omega)|$ if a quadratic chirp $\delta\omega_c$ = 159 MHz is added to the frequency modulation of the linearly chirped incident pulse. The spectrum is highly asymmetric about ω_o with a slowly decaying amplitude at higher frequencies. Considerable structure is evident in the spectral envelope for frequencies near ω_o. The peak near $(\omega-\omega_o)/2\pi$ = -62 MHz corresponds to frequencies in the leading edge of the pulse at $t \simeq -0.5T$ in Figure 2. The instantaneous carrier frequency is changing more slowly with time, as evidenced by the nearly horizontal slope of the dashed curve in Figure 2 for this portion of the incident pulse. Therefore, it is expected that these initial carrier frequencies are

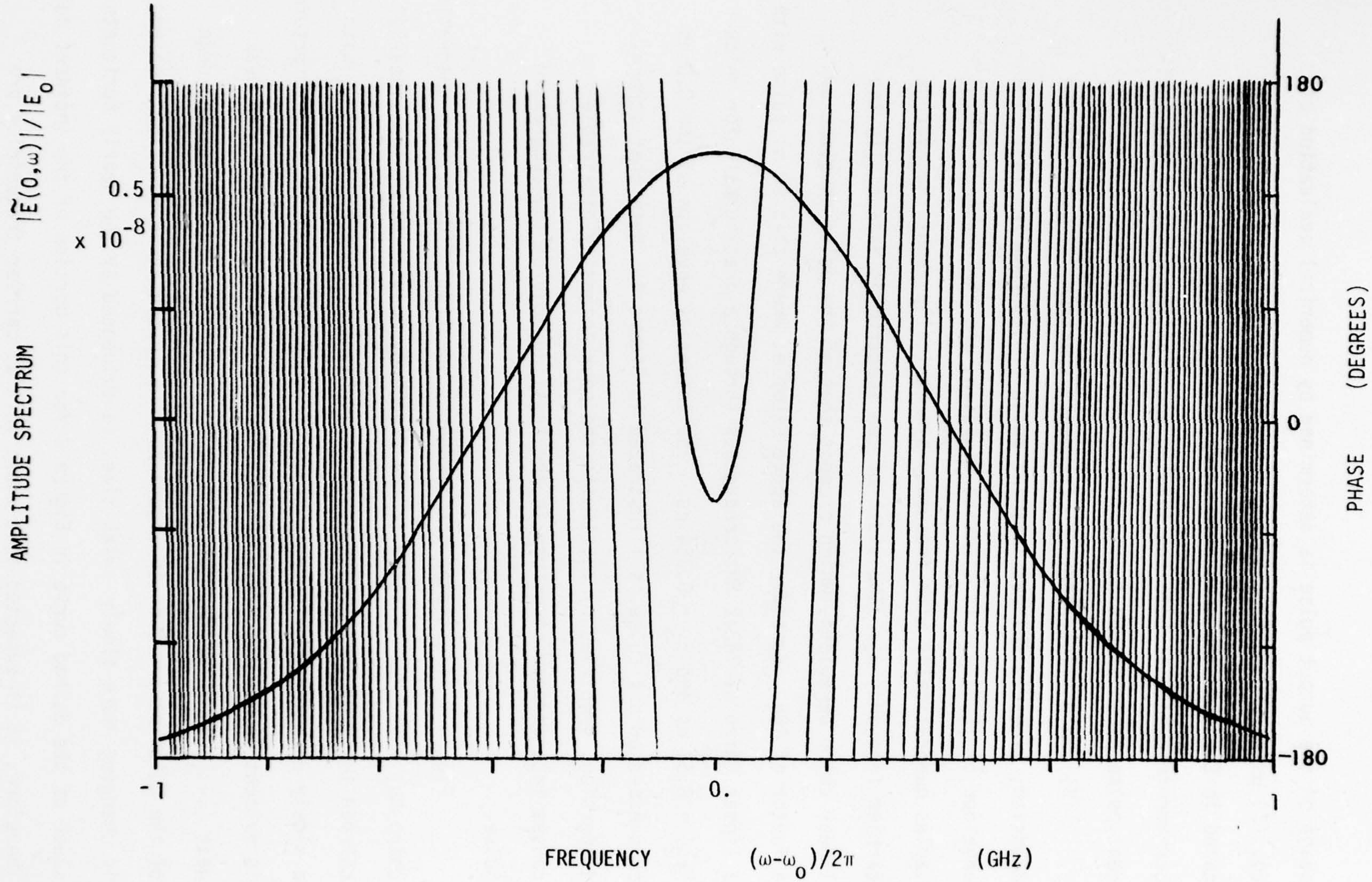

Figure III-3. Amplitude spectrum and phase of a linearly chirped, 80 ns wide pulse with linear chirp bandwidth $\delta\omega_m/2\pi$ = 768.2 MHz.

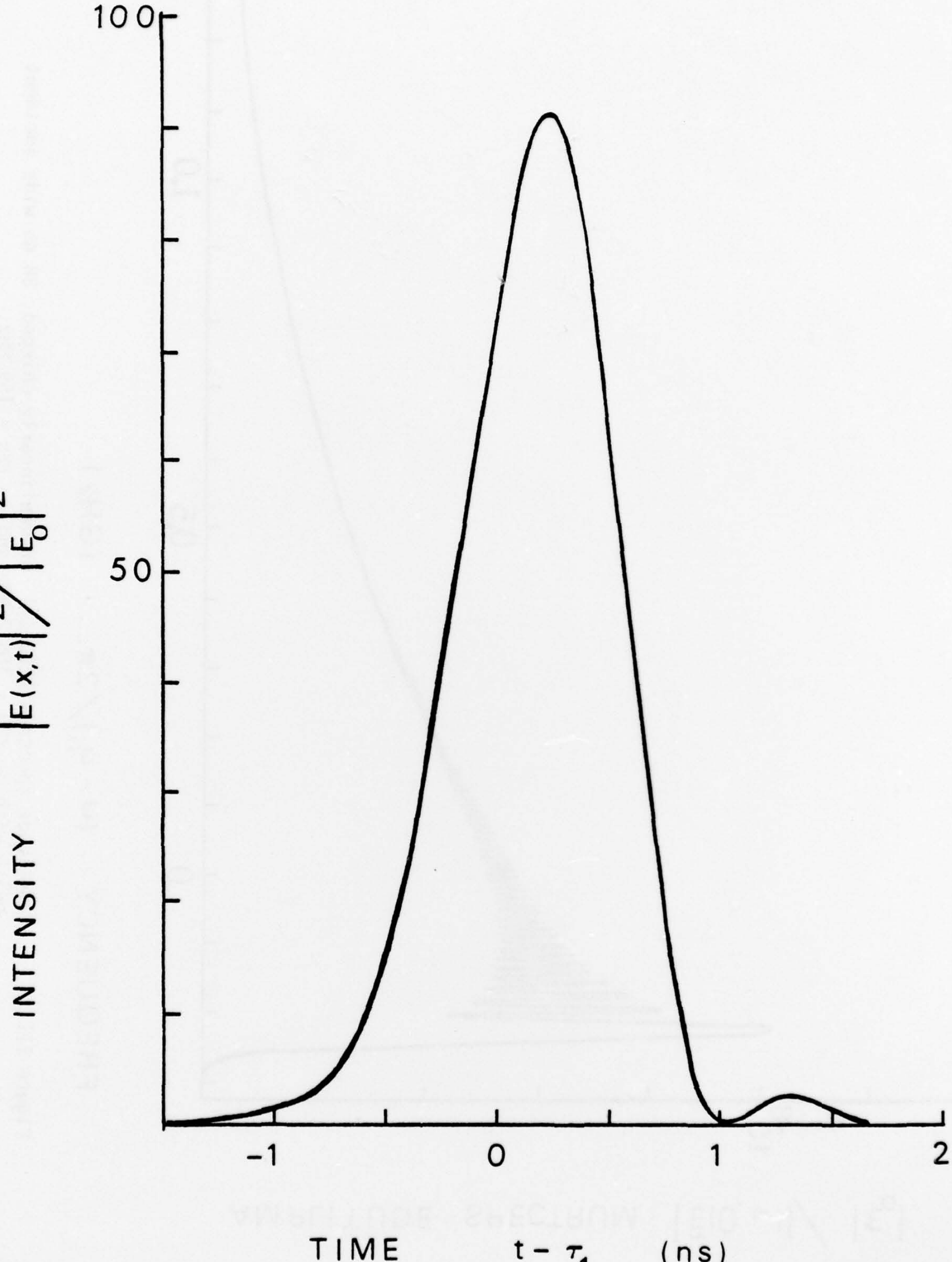

Figure III-4. Intensity envelope $|E(x,t)|^2$ of the linearly chirped pulse after propagating through a strongly dispersive medium for which $|\tau_2|$ = 8.54 ns and τ_3 = 0.384 ns.

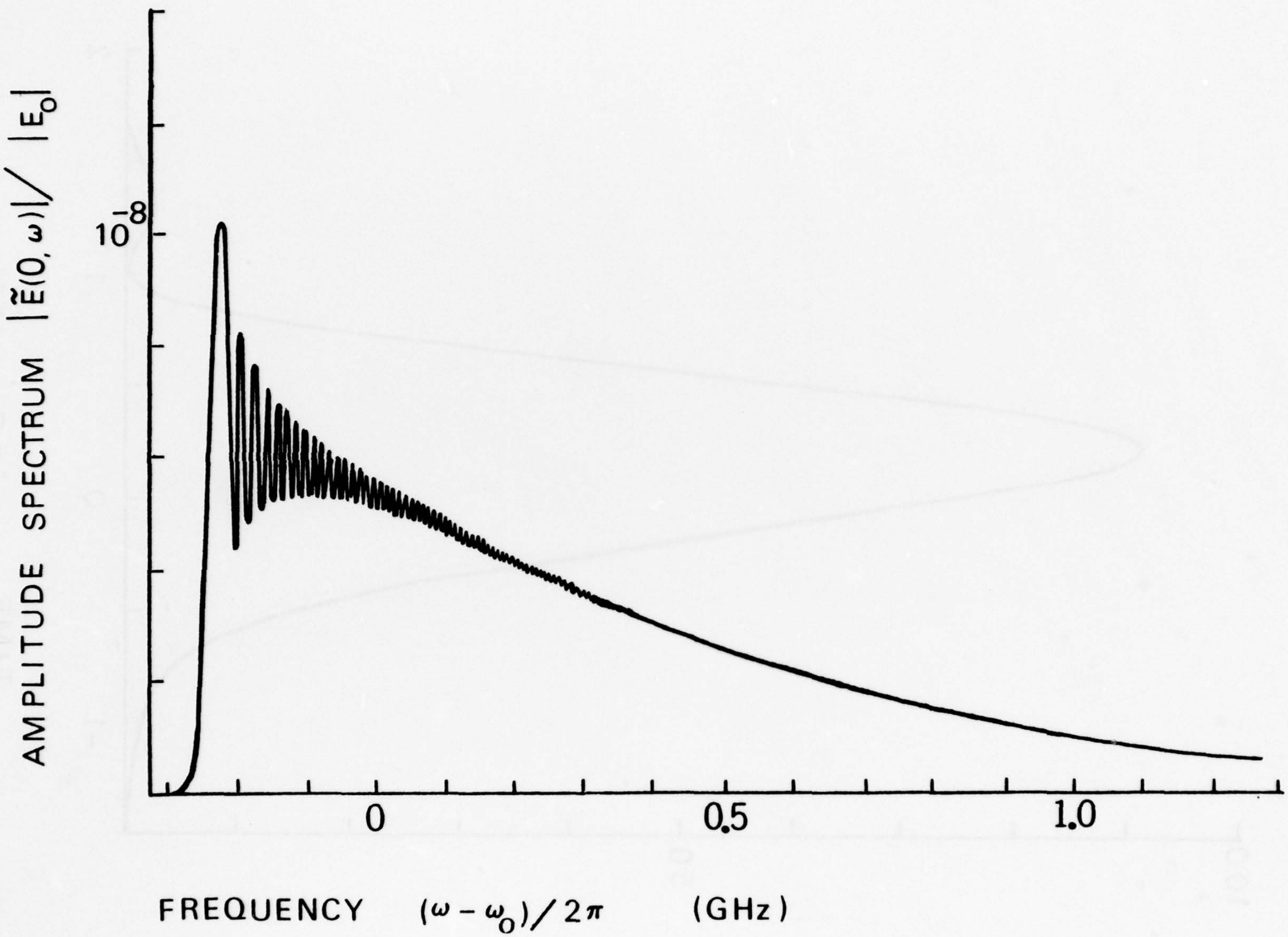

Figure III-5. Amplitude spectrum $|\tilde{E}(0,\omega)|$ of nonlinearly chirped, 88 ns wide incident pulse with $\delta\omega_m/2\pi$ = 768.2 MHz and $\delta\omega_c/2\pi$ = 159 MHz.

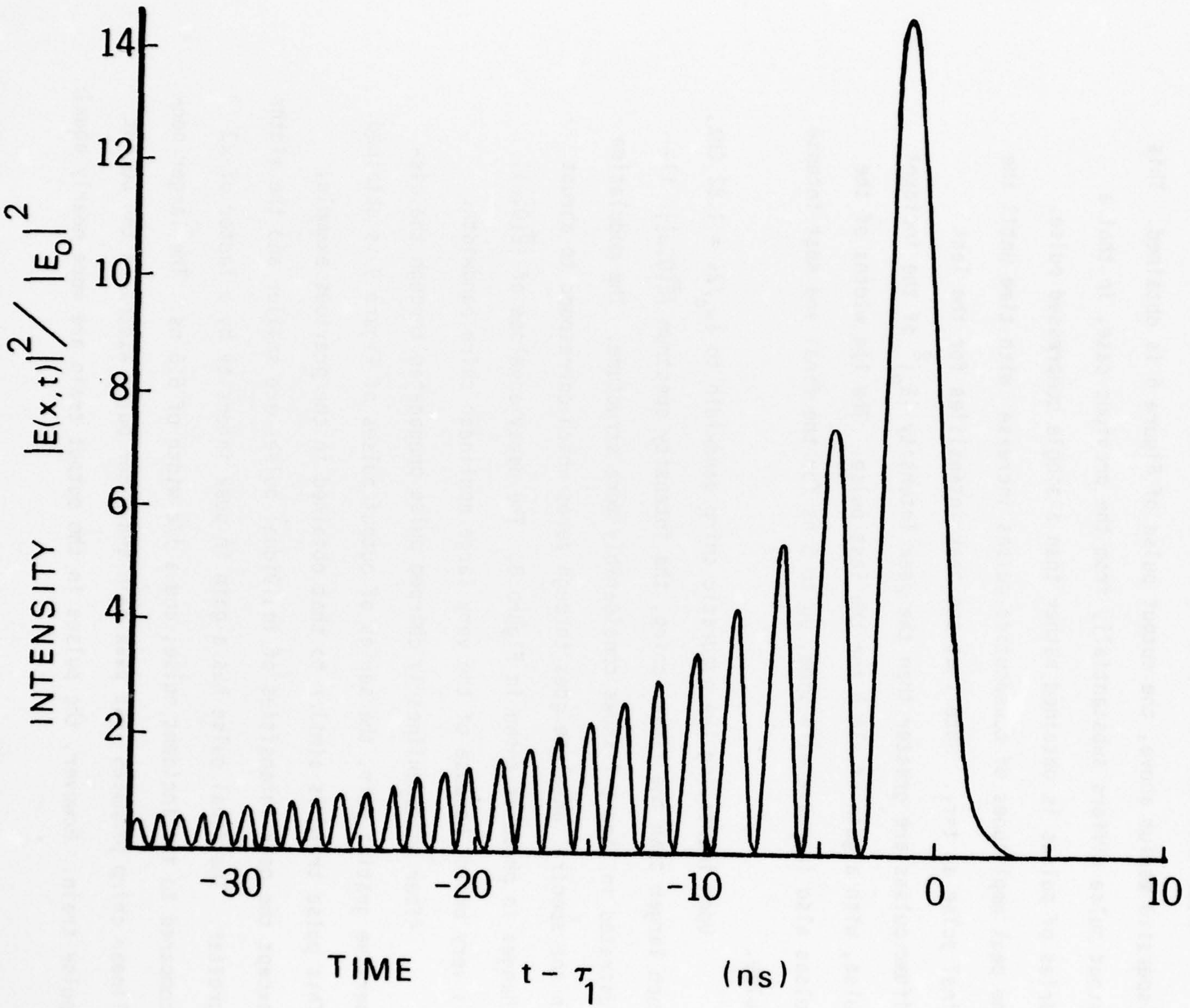

Figure III-6. Intensity envelope of compressed pulse for which $T = 88$ ns, $\delta\omega_m/2\pi = 768.2$ MHz and $\delta\omega_c/2\pi = 159$ MHz, after propagating through a strongly dispersive medium for which $|\tau_2| = 8.54$ ns and $\tau_3 = 0.384$ ns.

emphasized in the frequency spectrum $|\tilde{E}(0,\omega)|$ of Figure 5. The structure in the envelope of $|\tilde{E}(0,\omega)|$ is of limited depth, and becomes a nearly smooth exponential decay at higher frequencies.

After this nonlinearly chirped pulse propagates through the dispersive medium above, the output pulse of Figure 6 is obtained. This output pulse differs substantially from the previous case, in that a series of pulses is obtained rather than a single compressed pulse. The peak amplitudes of consecutive pulses increase with time until the final pulse at $t \approx \tau_1$. Note that the peak intensities for the last fifteen pulses are greater than the peak intensity $|E_0|^2$ of the incident pulse, with a gain of x14.5 for the last pulse. The 1/e widths of the pulses also increase with time, up to 3 ns for the final and most intense pulse.

Upon increasing the quadratic chirp bandwidth to $\delta\omega_c/2\pi$ = 1.82 GHz, much larger than the linear chirp, the intensity spectrum $|\tilde{E}(0,\omega)|$ illustrated in Figure 7 shows considerably more structure. The modulation in the spectral envelope goes through zeros which correspond to abrupt changes in phase as shown in Figure 8. The decay envelope of $|\tilde{E}(0,\omega)|$ is very broad, because of the very large nonlinear chirp bandwidth.

After this nonlinearly chirped pulse propagates through the dispersive grating pair, the series of output pulses of Figure 9 is obtained. This pulse train is similar to that obtained in the previous example, except the peak intensities of individual pulses are smaller and the widths greater. The final pulse has a gain in peak intensity by a factor of x3 compared to the incident pulse, and a 1/e width of 6.5 ns. The larger nonlinear chirp produces less peak gain and wider pulse widths in the output pulse train. However, the pulses in the output train are more nearly equal

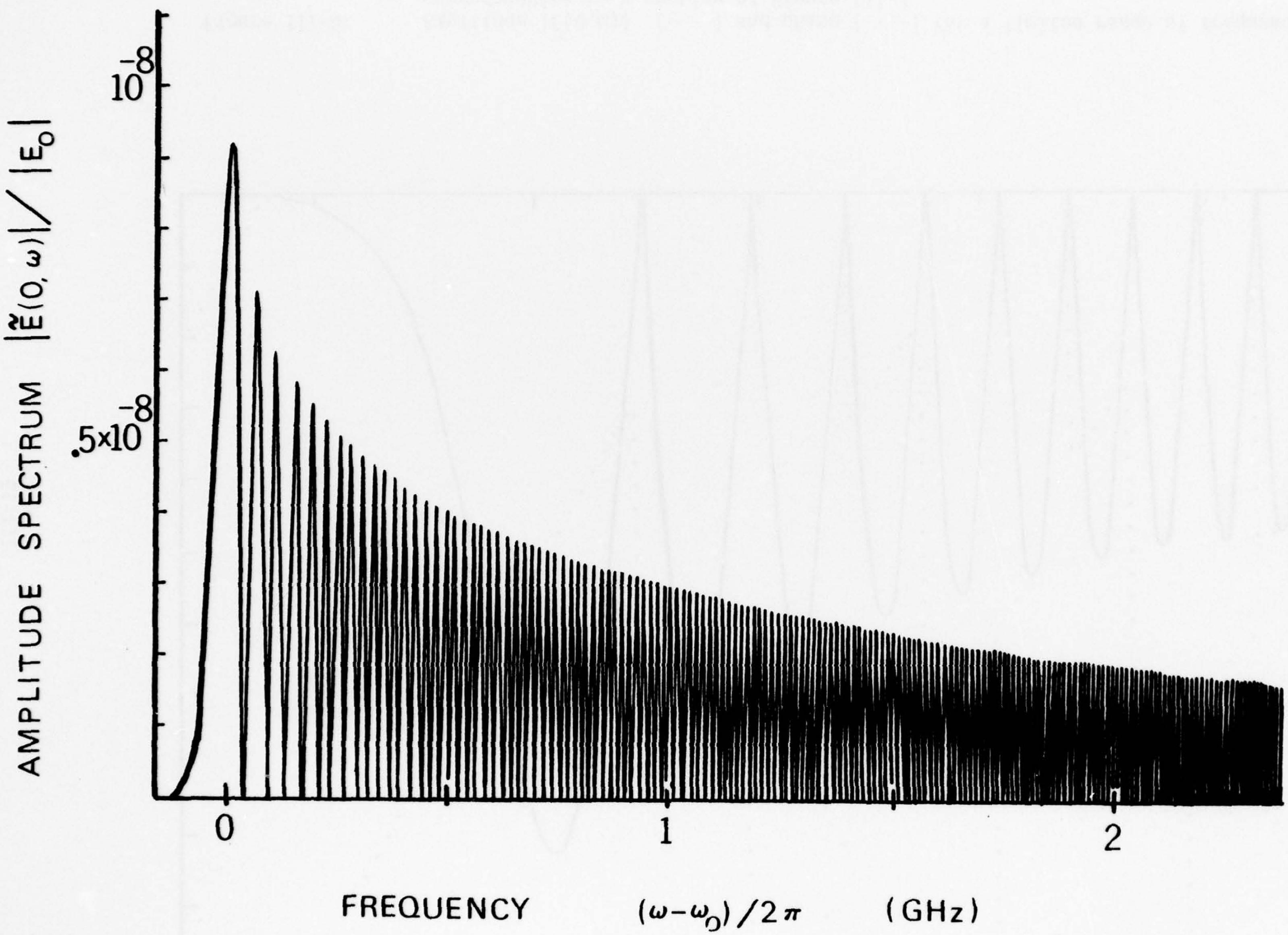

Figure III-7. Amplitude spectrum $|\tilde{E}(0,\omega)|$ of nonlinearly chirped 88 ns wide incident pulse with $\delta\omega_m/2\pi$ = 768.2 MHz and $\delta\omega_c/2\pi$ = 1.82 GHz.

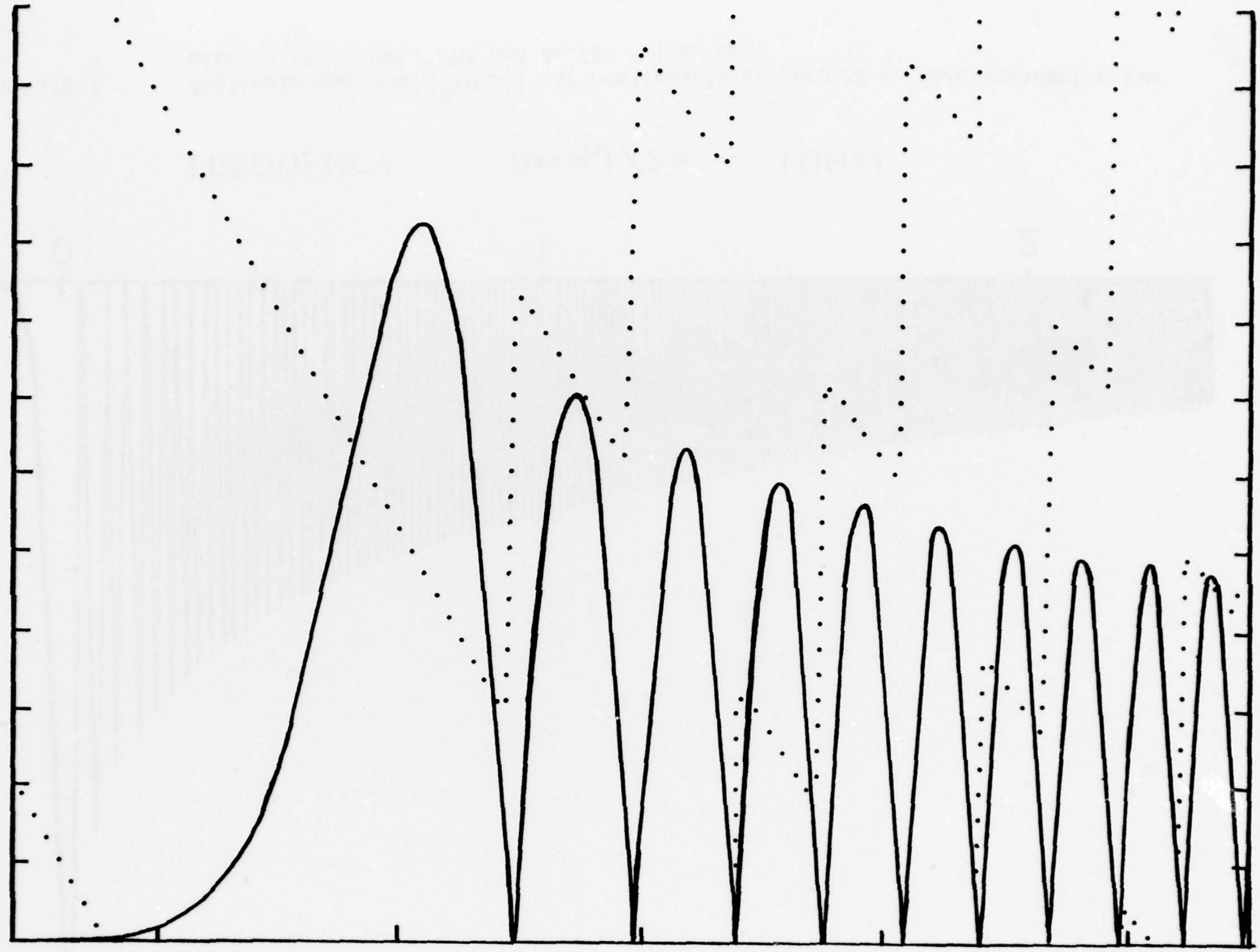

Figure III-8. Amplitude $|\tilde{E}(0,\omega)|$ (——) and phase (····) for a limited range of frequencies corresponding to a portion of Figure III-7.

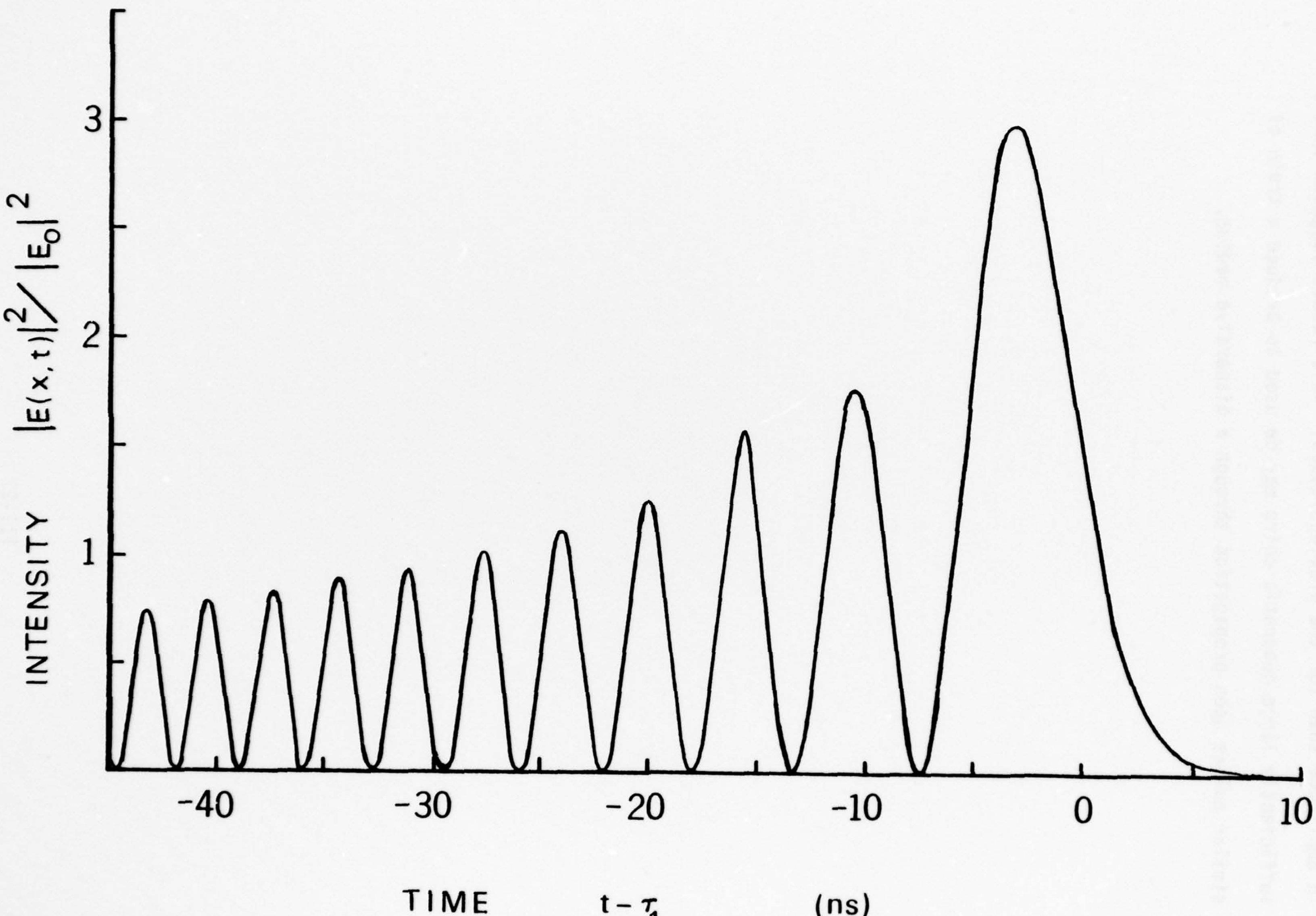

Figure III-9. Intensity envelope of compressed pulse for which T = 88 ns, $\delta\omega_m/2\pi$ = 768.2 MHz and $\delta\omega_c/2\pi$ = 1.82 GHz after propagating through a strongly dispersive medium for which $|\tau_2|$ = 8.54 ns and τ_3 = 0.384 ns.

in amplitude than for the previous cases. This result suggests that a sufficiently large quadratic chirp may be used to produce a train of similar pulses upon propagation through a dispersive medium.

III-C. CONCLUSIONS AND RECOMMENDATIONS

An analytic solution has been obtained in closed form for the frequency spectrum of a pulse which contains both linear and quadratic variation of instantaneous carrier frequency with time. This solution greatly facilitates computer calculations of the output pulse obtained by propagating the nonlinearly chirped pulse through a strongly dispersive medium, such as can be formed by a grating pair. If the linear chirp bandwidth is maintained at the value required for optimum compression with $\delta\omega_c=0$, then addition of the quadratic frequency chirp to the incident pulse causes the output to consist of a train of pulses of ascending peak intensities. Compression gain and narrowing are still obtained for a number of these output pulses, although to a lesser extent than for the case of only a linear chirp. These results indicate that a sufficiently large quadratic frequency chirp may be used to generate a pulse train from a single broad incident pulse. The envelope of peak intensities in this pulse train varies more slowly for a larger quadratic chirp bandwidth.

Further studies should include cases where the linear chirp bandwidth is smaller than the optimum value required to compress a linearly chirped pulse to the narrowest possible width. For this case, the additional bandwidth in the nonlinear chirp would reinforce or "make up for" the inadequate bandwidth of the linear chirp in the trailing edge of the incident pulse, but would "oppose" the sense of time variation of the linear chirp in the leading edge.

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